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M 564.01: Topics in Analysis

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Math 564 - Topics in Analysis, "Graph C^*-algebras"

Spring 2018

Course Instructor: Dr. Elizabeth Gillaspy

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Lecture hours and Venue: MWF 12:00 p.m. - 12:50 a.m., MATH 305

Office Hours: Monday 11 a.m. - 12 p.m.; Tuesday 12 - 1 p.m.; and by appointment.

Textbook: Iain Raeburn, *Graph algebras.*

1 Course Contents

C^*-algebras were first invented by physicists, as a model for quantum mechanics; they have since turned out to give useful information about groups, and other structures from topology and algebra. However, as befits an analytic object, C^*-algebras are defined by taking a bunch of limits. You don't necessarily know what those limits are, just that they exist, so it can be hard to get a good handle on a random C^*-algebra.

Graph C^*-algebras were invented to fill in this gap. Many natural questions about C^*-algebras can be answered, in the case of graph C^*-algebras, just by looking at the graph you started with! So, finding graphs with certain properties can help you understand whether a C^*-algebra with certain other (related) properties could exist. However, these properties of graphs that we C^*-algebraists care about are not always the same ones that graph theorists care about.

My hope is that this Graph C^*-algebras course will interest some graph theorists in new questions about graphs, arising from C^*-algebra theory; introduce people from many backgrounds to C^*-algebras; and hopefully find links between your interests in algebra, topology, and/or physics with graphs and C^*-algebras.

Because of all this, I will try to assume as little background as possible for M 564. I won't assume everyone has taken functional analysis, for example. I would ask that if you're taking the course, you've taken at least one 400-level or graduate level analysis course – but there won't be any formal prerequisites.

2 Course Structure

Your grade in this course will be based on:

- Reading assignments/Discussion posts (5%)
- Presentations (15%)
- Homework (80%)

2.1 Reading assignments/Discussion posts

In every class I teach, I want students to improve their ability to communicate math, not just to enhance their mathematical knowledge. One aspect of this is improving your ability to read math. To that end, I will usually ask you to read a section of the textbook before class, and post a comment in the relevant Discussion forum in Moodle by **10 AM** the morning before class. I don't expect you to be able to understand everything in the textbook at the beginning of the semester – this is the role of the Discussion forums, for you to tell me what you didn't understand from the textbook and what you want to focus on in class that day. However, I do expect you to put serious effort into the reading assignments, so that your ability to learn math from the textbook will improve over the course of the semester. We will spend time in class discussing strategies for reading a math textbook; one aspect of this is to **budget 10 minutes per page** for the reading assignments.

The discussion posts are a means for you to synthesize what you've learned by reading the textbook, connect the material to previous sections, and let me know what you still need some help with. Your comments in the Discussion forums will be graded on a scale of 0 to 2, based on how well you convince me that you have read the section and thought about it, and I will drop your lowest 5 scores.

For example:

- You will receive a 0 if you don't post any comment.
- A comment like "Everything makes sense, I have no questions" will earn a score of 0.

  (Even if everything makes sense, I'm sure you can find something more specific to say! How does this section connect with other sections? What did you think was interesting? Was there any part of the section that didn't make sense at first? What was it that eventually made that part make sense?)
- If your comment asks a question that indicates to me that you only skimmed the textbook, you will earn a score of 1.

  For example: "What does it mean to say that two projections are mutually orthogonal?"

  (This is explained in words on page 7 of the textbook.)
- A comment such as "I don't understand why, in Example 1.11, we have $\dim(S_f\mathcal{H}) = \dim(P_w\mathcal{H})$. Is it related to the fact that S_f is an isometry on its initial space?" will earn a score of 2.

  This question demonstrates that the author read the section and thought about what s/he read.

2.2 Presentations

To enhance the relevance of M 564 to your interests, I will ask each of you to give a short presentation (20-25 minutes) about a topic of interest to you that relates to Graph C^*-algebras. You can base this presentation on a section of the textbook that we haven't covered in class, or on material from another textbook or a paper, or even material you've learned in another course if it's relevant and will be new to most of the class.

Please let me know as soon as you have an idea for a presentation topic. Then we can fine-tune the choice of topic and source materials together, and schedule your presentation so that it fits well with the arc of the course and with your own schedule. At the very latest, you should **meet with me before April 6** to confirm your presentation topic and schedule your presentation.

2.3 Homework

The majority of your grade will be based on the weekly written homework assignments, which will be due on Wednesdays at the beginning of class. Each of the 13 homework assignments will be graded out of 20 points, but the total number of points in the "Homework" category of your grade will be 200, to give you a buffer for busy weeks.

I encourage you to discuss the homework problems together, but **you must write up your solutions on your own**, and acknowledge, in your write-ups, any sources (human or otherwise) that you consulted, other than me or the course textbook. During discussions with classmates (or with me) about the homework problems, feel free to take a few notes, but you should not write up the solutions during this discussion period. Waiting to write up the solutions until you're in the privacy of your home or office confirms that the solutions you write up reflect your own understanding of the problem.

Indeed, when writing up your homework, please **consult your notes, and any other resources, as minimally as possible.** Before writing the proof, think through the argument. How do all the pieces fit together? What's the best order to explain things? Then, once you start writing, do your best to write the solution without reference to your notes or the textbook. If you have to look up a reference more than 2 or 3 times during the writing, you don't fully understand the solution yet. If this happens, I suggest you stop working on the final draft; work through again (using notes) how the argument should go; and take a break, perhaps by moving on to another problem. When you come back to the tricky problem later, you should have a better, deeper understanding of how the proof works.

Failure to follow these guidelines may result in homework solutions that are uncomfortably close to plagiarism. Plagiarism (copying solutions from a source other than your own brain) constitutes academic misconduct, and will result in a loss of credit for the assignment and/or disciplinary sanctions.

3 Course Policies

3.1 Asking and Answering Questions

It's important to me that our class is paced for everyone, not necessarily the pace of the most vocal students. I want to make sure that everyone's questions are heard. So, if you have a question, feel free to ask it at any time; if I have a question for the class, I will call on someone randomly. Please feel free to say "I don't know;" that tells me that I need to try a different explanation. Almost always, if you're confused, there are at least one or two other people in the class with the same confusion!

3.2 Student Classroom and Course-Related Behavior

University policy requires that all of us in the classroom treat each other with respect, and refrain from behavior that will disrupt the educational process. In particular, I expect that you will **focus your attention on M 564 during class time!** For example, please refrain from cell phone use (including texting), computer use (unless you take notes on your computer), or working on homework for another class. I know from experience how easy it is to think "I know this material, it's a good time to take 10 seconds and check my email" – only to surface a full minute later, having missed something important.

In my turn, I promise to treat all of you with respect. For example, if you would prefer me to call you by a different name, or gender pronoun, than is listed on the course roster, please let me know and I'll be happy to oblige.

3.3 Due dates and late work

Unless stated otherwise, all assignments are due at the beginning of class on the day on which they are due. Late work will receive no credit. Turning in an assignment 5 minutes after the start of class is acceptable (unless it becomes a habit); handing in an assignment 20 minutes late is **not**.

3.4 Disability Modifications

The University of Montana assures equal access to instruction through collaboration between students with disabilities, instructors, and Disability Services for Students. If you have a disability that adversely affects your academic performance, and you have not already registered with Disability Services, please contact Disability Services in Lommasson Center 154 or 406.243.2243. I will work with you and Disability Services to provide an appropriate modification.

3.5 Academic Honesty

All students must practice academic honesty. Academic misconduct is subject to an academic penalty by the course instructor and/or a disciplinary sanction by the University.

3.6 Religious Holidays and Absences from Classes and/or Exams

If a religious observance conflicts with a scheduled exam or other course activity, please let me know **during the first two weeks of the semester** so that we can make alternate arrangements.

3.7 Student Conduct Code

All students need to be familiar with the Student Conduct Code. You can find it in the "A to Z" index on the UM home page.

In particular, discrimination and harassment are not tolerated at the University of Montana. If you feel that you have been subjected to discriminatory or harassing behavior, I'm very sorry to hear it; please contact the Office of Equal Opportunity and Affirmative Action at 243-5710 or `http://www.umt.edu/eo/equalop/harassment.php` for help in addressing the situation. You can also report the discrimination or harassment to me or to another faculty member you trust.